Esercizi

Esercizio 1 $\qquad \dfrac{x}{x+2}-\dfrac{3-x}{x^2-4}>\dfrac{x+1}{x-2}$ $\qquad$ 1

1) determino il C.E. $\begin{cases} x+2\neq 0 \\ x^2-4\neq 0 \\ x-2\neq 0 \end{cases} \iff \begin{cases} x+2\neq 0 \\ x-2\neq 0 \end{cases} \iff \boxed{x\neq -2 \wedge x\neq 2}$

$\boxed{C.E.\ x\neq \pm 2}$

2) cerco di riportare la frazione in "forma normale"

$$\frac{x}{x+2}-\frac{3-x}{x^2-4}>\frac{x+1}{x-2} \iff \frac{x}{x+2}+\frac{x-3}{(x-2)(x+2)}-\frac{x+1}{x-2}>0$$

$$\iff \frac{x(x-2)+(x-3)-(x+1)(x+2)}{(x-2)(x+2)}>0 \iff \frac{x^2-2x+x-3-(x^2+3x+2)}{x^2-4}$$

$$\iff \frac{x^2-x-3-x^2-3x-2}{(x-2)(x+2)}>0 \iff \frac{-4x-5}{(x+2)(x-2)}>0 \iff \frac{4x+5}{(x+2)(x-2)}<0$$

Studio $\boxed{\dfrac{4x+5}{(x+2)(x-2)}<0}$

1) $N>0 \to 4x+5>0 \iff \boxed{x>-5/4}$

2) $D>0 \to (x+2)(x-2)>0 \iff \boxed{x<-2 \vee x>2}$

3) adesso devo fare il prodotto dei segni

	-2	$-5/4$	2	
N	$-$	$-$	$+$	$+$
D	$+$	$-$	$-$	$+$
	$-$	$+$	$-$	$+$

quindi la soluzione

$$\boxed{S=(-\infty,-2)\cup\left(-\frac{5}{4},2\right)}$$

Insieme delle soluzioni della disequazione

$$\frac{x^2-3x+2}{x^2-4x}<0$$

(2)

1) C.E. $x^2-4x \neq 0$

$\Updownarrow$

$x(x-4)\neq 0$

quindi $\boxed{x\neq 0 \wedge x\neq 4}$ C.E

2) studio $N>0 \rightarrow x^2-3x+2>0$ $\quad a=1>0$ concorde $\quad \Delta=9-8=1>0$

$x_{1,2}=\frac{3\pm 1}{2} <^{\frac{4}{2}=2}_{\frac{2}{2}=1}$

sol. esterne

$x_1\ 1 \quad x_2\ 2$

$S_N=\boxed{x<1 \vee x>2}$

3) studio $D>0 \rightarrow x^2-4x>0$ $\quad a=1>0$ concorde

$\Delta>0 \quad x_1=0,\ x_2=4$

sol. esterne

$0 \quad 4$

$x<0 \vee x>4$

4) Prodotto dei segni

		0	1	2	4
N	+	+	−	+	+
D	+	−	−	−	+
	+	−	+	−	+

$\boxed{(0,1)\vee(2,4)=S}$

Insieme delle soluzioni della disequazione

③

$$\frac{2sen(x)-1}{2\cos(x)+1} > 0 \qquad C.E.\ \cos(x) \neq -\tfrac{1}{2}$$

sia $f(x) = \frac{2sen(x)-1}{2\cos(x)+1}$ allora $f(x+2\pi) = \frac{2sen(x+2\pi)-1}{2\cos(x+2\pi)+1}$

poichè $sen(x+2\pi) = sen(x)$ e $\cos(x+2\pi) = \cos(x)$

allora $f(x+2\pi) = f(x)$ quindi $f(x)$ è una funzione periodica di periodo $\underline{T = 2\pi}$

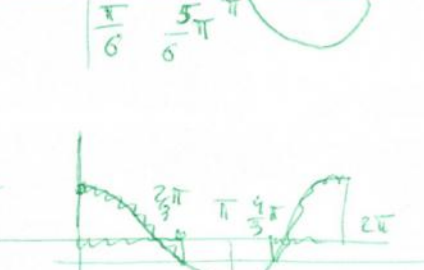
posso quindi limitarmi a studiare la disequazione nell'intervallo $[0, 2\pi]$

$\cos(x) \neq -\tfrac{1}{2} \iff \boxed{x \neq \frac{4}{3}\pi \wedge x \neq \frac{2}{3}\pi}$ C.E in $[0, 2\pi]$

considero $N>0 \Rightarrow sen(x) > \tfrac{1}{2}$

$\Updownarrow$

$\boxed{\frac{\pi}{6} < x < \frac{5}{6}\pi}$



considero $D>0 \Rightarrow \cos(x) > -\tfrac{1}{2}$

$\Updownarrow$

$\boxed{0 < x < \frac{2}{3}\pi \vee \frac{4}{3}\pi < x < 2\pi}$

4) Studio del segno

	0	$\frac{\pi}{6}$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	$\frac{4}{3}\pi$	2π
N	$-$	$+$	$+$	$-$	$-$	
D	$+$	$+$	$-$	$-$	$+$	
S	$-$	$+$	$-$	$+$	$-$	

$$S_p = \left(\frac{\pi}{6}, \frac{2}{3}\pi\right) \cup \left(\frac{5}{6}\pi, \frac{4}{3}\pi\right)$$

$$\boxed{S = S_p + 2k\pi, \ k \in \mathbb{Z}}$$ Insieme soluzioni dello disequazione

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$$\frac{\tan(x)}{\tan^2(x) - 1} < 0 \qquad \text{pongo } w = \tan(x)$$

e studio $\frac{w}{w^2-1} < 0$ $\boxed{w \neq \pm 1 \ \ C.E.}$

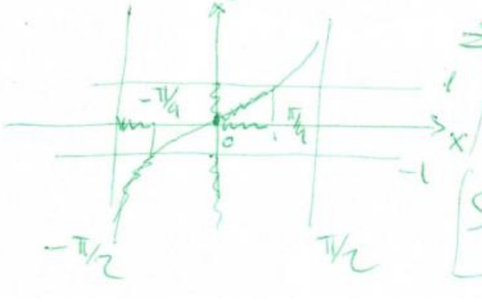
$N > 0 \Rightarrow w > 0$

$D > 0 \Rightarrow w^2 - 1 > 0 \Leftrightarrow w < -1 \vee w > 1$

	-1	0	1	
N	$-$	$-$	$+$	$+$
D	$+$	$-$	$-$	$+$
	$-$	$+$	$-$	$+$

$S_w = -1 > w \vee 0 < w < 1$

cioè $\tan(x) < -1 \vee 0 < \tan(x) < 1$



$\Rightarrow S_p = \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(0, \frac{\pi}{4}\right)$

$$\boxed{S = S_p + k\pi, \ k \in \mathbb{Z}}$$